A review of hydrodynamical models for semiconductors: asymptotic behavior

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— Dedicated to Constantine Dafermos on his 60th birthday

Abstract. We review recent results on the hydrodynamical model for semiconductors. The derivation of the mathematical model from the semi-classical Boltzmann equation in terms of the moment method is performed, and the mathematical analysis of the asymptotic behavior of both classical solutions and entropy weak solutions is given on spatially bounded domain or whole space.

Keywords: hydrodynamical models, semiconductors, asymptotic behavior.

1 Introduction

The topic of semiconductor device modeling has an important place in research areas like solid state physics, applied and computational mathematics. It ranges from kinetic transport equations for charge carriers (electrons and holes) to fluid dynamical models. The kinetic equations may be quantum mechanical, semiclassical or classical models. The main principle of classical model in the description of the motion of particle ensembles is Newton's second law applied to ballistic transport and scattering events of the charge currencies. As crystal lattice effects are taken into consideration, we obtain semiclassical and, on other shorter scales, quantum models. The typical semiclassical phase space model is the Boltzmann equation. Approximating the solutions of this kinetic transport model by performing scaling limits, we obtain fluid dynamical models. In particular, the Hilbert expansion of the Boltzmann equation leads to the drift-diffusion or Van

Roosbroeck model [68, 69, 71] based on the assumptions of low carrier densities and small fields. To describe high field phenomena, the hydrodynamical model of semiconductor was derived from the Boltzmann equation by moment method based on the shifted Maxwellian ansatz for the equilibrium phase space distribution.

Here we are interested in the mathematical theory of hydrodynamical equations themselves. These equations are typically considered on the whole position space or on bounded domains with appropriate boundary conditions [2, 73]. The full hydrodynamical model is a quasilinear hyperbolic–parabolic–elliptic system of balance laws. This system for the particle densities, the particle velocities, the temperature and the electric potential makes it complicated to understand the qualitative behavior of solutions. Recently, many efforts were made to understand the influence of the coupling effects on the solutions, such as existence and regularly, large time behavior and zero relaxation limit, etc.

The effect of the Poisson (electric) potential coupling is smoothing and, also, it decisively affects the stationary states of the hydrodynamical equations. Usually the stationary solution, especially the particle density, is not a constant state. In fact, it solves a quasilinear second order elliptic equation of divergence form (for the particle density) dependent on the current density. Degond and Markowich [16] first considered the well-posedness of subsonic stationary solutions on one-dimensional intervals for small current density and electrostatic potential. The key idea of reducing to a second order elliptic equation is applied to establish the existence result. The current-voltage relationship was used to obtain the uniqueness for given small current density and potential. Afterwards, related results on the existence of subsonic stationary solution were obtained in multidimensional cases [61, 17, 76]. The existence of transonic stationary solutions was also proven in one-dimension [26, 25] and in the multi-dimensional case by Gamba-Morawetz [27] using the artificial viscosity method; and proven in one space dimension in terms of a phase plane analysis [62, 63].

Due to the damping (relaxation) dissipations of momentum and heat flux and the Poisson potential coupling, we expect to maintain regularity of dynamical solutions in subsonic regions. The first results on dynamical solutions by Luo-Natalini-Xin [55] showed that global classical (subsonic) solutions tend exponentially to the stationary equilibrium solution of the drift-diffusion equation in one-dimensional case. This is based on the fact that frictional damping causes diffusive phenomena [35, 36]. A further study by Marcati–Mei [57] showed that general steady-state solutions of hydrodynamical equations are locally exponentially stable stationary state. Related results were also established for the

one-dimensional full hydrodynamical equations (with the temperature equation) [32, 31, 37, 1] and for the multi-dimensional hydrodynamical equations [38].

When boundary effects are taken into consideration on bounded position domains, the dynamics of solutions become more complex. Under the assumption of zero-current density on the boundary, Chen-Jerome-Zhang [12] proved the convergence of classical solutions to a constant state (as $t \to \infty$) based on the local existence result [78]; while Hsiao and Yang [40] showed the convergence to the unique steady-state solution of the drift-diffusion equations. Li-Markowich-Mei [50] proved the convergence to subsonic stationary states of hydrodynamical equations with density and potential boundary conditions, which means that in the subsonic regime there is a unique classical stationary solution for small current density and potential. This solution is exponentially stable in time for small perturbations.

On the other hand, due to the strong hyperbolicity it was proven [14] that smooth solutions may cease to exit in finite time. Even the heat conduction can't prevent the formation of singularities (of smooth solutions for initially large data). Thus, it is natural to consider the existence and asymptotic of weak solutions, too. The global existence of large entropy weak solutions was obtained in BV-framework by Poupaud-Rascle-Vila [70]. The global existence of L^∞ entropy solutions and/or zero relaxation limit were shown by Marcati–Natalini [58, 59], Chen–Wang [13], Hsiao–Zhang [41, 42], Jüngel–Peng [47, 48], Zhang [80, 81], Gasser–Natalini [30] and [77, 24, 44] in the compensated compactness framework developed by DiPerna [21, 22], Ding–Chen–Luo [18, 19, 10, 20], and Lions-Perthame-Sougandis-Tadmor [53, 54]. The reader is refered to [65] for more references. It is also important to investigate the global existence and asymptotic behavior of entropy weak solutions. However, few results are known about the influence of the Poisson coupling and damping dissipation on the asymptotic behaviors of weak solutions. The only known one is on subsonic shock solutions by Li-Markowich-Mei [51] where a detailed investigation on shock (rarefaction) discontinuities was made.

Furthermore, though the mathematical analysis on the existence of global solution was considered extensively [67, 65,33, 29], the investigation of the dynamical behavior of solutions of a bipolar hydrodynamical model is far from well understood. Up to now, Hattori and Zhu [33] proved the asymptotic convergence of classical solutions to bipolar hydrodynamical steady state model with re-combination. While Gasser–Hsiao–Li [29] observed that the frictional damping is strong enough to cause nonlinear diffusive phenomena of the bipolar hydrodynamical equations without background ions ($\mathcal{C}(x) = 0$) in the sense that

both the electrons and the holes densities tend to the same diffusive wave at an algebraic rate, and the electric field decays to zero exponentially. However, few results are known on the understanding of boundary effects.

This paper is arranged as follows. The derivation of hydrodynamical models for semiconductors from the semi-classical Boltzmann equation in terms of the moment method is performed in section 2. In section 3, we state some results on the large time behavior of subsonic solutions (for classical solutions in section 3.1 and for subsonic shock solutions in section 3.2) of the unipolar isentropic hydrodynamical model for semiconductors. In section 4, we review the results of subsonic solutions of the bipolar isentropic hydrodynamical model. There a nonlinear diffusive wave is the asymptotic state of both densities.

2   Derivation of hydrodynamical models (HD)

The hydrodynamical model for semiconductors can be derived from the Boltzmann equation by applying the moment method. The starting point is the kinetic transport equation, the Boltzmann equation, which describes the phase space motion of charge carriers (electrons or holes) in semiconductor crystals. Let $f = f(x, k, t)$ where $x \in \mathbb{R}^3$ denotes the position variable, $k \in \mathcal{B}$ denotes the wave vector, and $\mathcal{B}$ the Brillouin zone related to the crystal lattice [8]. Here we take $\mathcal{B} = \mathbb{R}^3$ [64] for the role of simplicity. With $t > 0$ the time variable, the unipolar Boltzmann equation for semiconductors takes the form [64, 45]

$$\partial_t f + v(k)\nabla_x f + \frac{q}{\hbar}\nabla_x V \cdot \nabla_k f = Q(f, f), \ (x, k, t) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}_+, \qquad (2.1)$$

$$f(x, k, 0) = f_0(x, k), \ (x, k) \in \mathbb{R}^3 \times \mathbb{R}^3. \qquad (2.2)$$

Here, $q > 0$ is the elementary charge and $\hbar$ the reduced Planck constant. $v(k) = (1/\hbar)\nabla_k \varepsilon(k)$ is the mean electron velocity and $\varepsilon(k)$ is the energy-wave vector function. The collision operator $Q(f, f)$ is supposed to model the short range interactions of the electrons with crystal impurities and photons. Electron interaction is neglected here [64]. It takes the form

$$Q(f, f) = \int \psi(x, v, v')(Mf' - M'f)dv', \qquad (2.3)$$

where $\psi(x, v, v')$, a symmetric function in v and v', is the scattering rate, and the Maxwellian is given by

$$M(v) = \left(\frac{m}{2\pi k_\mathcal{B} T}\right)^{3/2} \exp\left(\frac{-m|v|^2}{2k_\mathcal{B} T}\right). \qquad (2.4)$$

Here $k_{\mathcal{B}}$ is the Boltzmann constant. In the parabolic band approximation, the energy band function $\varepsilon(k)$ reads

$$\varepsilon(k) = E_c + \frac{\hbar^2}{2m}|k|^2, \tag{2.5}$$

where E_c denotes the conduction band minimum and m the effective masses of electrons. In this case, the velocity is given by

$$v(k) = \frac{\hbar}{m}k.$$

The electronic potential $V = V(x, t)$ is coupled self-consistently to the Poisson equation

$$\lambda^2 \Delta V = q(\rho - \mathcal{C}), \tag{2.6}$$

where λ denotes the semiconductor Debye length and $\mathcal{C} = \mathcal{C}(x)$ is the doping profile of the background charge ions. The electron density $\rho = \rho(x, t)$ is defined by

$$\rho = \int f\, dk. \tag{2.7}$$

To describe high field phenomena and submicronic semiconductor devices, it is more efficient to use hydrodynamical models. These models can be obtained by approximating the Boltzmann equation (2.1) by the moment method (eliminating the wave vector variable). The key idea is to derive equations for the moments of the distribution function. The j-th order moment of the distribution function f is defined as the tensor M_j of rank j given by

$$M_j^{i_1, \dots, i_j}(x, t) = \int v_{i_1} \cdots v_{i_j} f(x, k, t)dk, \ \ j \geq 1, \ 1 \leq i_1, \dots, i_j, \leq 3;$$

$$M_0(x, t) = \int f(x, k, t)dk.$$

The relevance of the moments lies in the fact that they are directly related to physical quantities. The first three moments corresponding to the multipliers 1, k, and $\frac{1}{2}|k|^2$ are, for instance,

$$M_0 = \rho \qquad\qquad \text{position space number density,}$$
$$M_1 = J = \rho u \qquad\qquad \text{current density,}$$

$$\frac{m}{2}\text{trace}(M_2) = \mathcal{E} \qquad\qquad \text{energy density.}$$

Here u denotes the electric velocity. The energy density can be written as the sum of a kinetic and a thermal contribution

$$\mathcal{E} = \rho\left(\frac{m}{2}|u|^2 + \frac{3}{2}\kappa_{\mathcal{B}}T_e\right). \tag{2.8}$$

The *displaced* (or *shifted*) Maxwellian ansatz for the distribution funtion reads

$$f(v) = \rho\left(\frac{m}{2\pi k_{\mathcal{B}}T_e}\right)^{3/2} \exp\left(\frac{-m|k-u|^2}{2k_{\mathcal{B}}T_e}\right), \tag{2.9}$$

in analogy to gas dynamics. ρ, u, and T_e are the free parameters depending on position and time. The Maxwellian function (2.9) can be used as an ansatz for the moment method with the three free parameters.

Multiplying the Boltzmann equation (2.1) with 1, k, and $\frac{1}{2}|k|^2$ respectively, integrating over the velocity space and using (2.9), we obtain a system of equations for the first three moments of the distribution function:

$$\partial_t \rho + \text{div}\,(\rho u) = 0, \tag{2.10}$$

$$\partial_t(\rho u) + \text{div}\,(\rho u \otimes u) + \frac{k_{\mathcal{B}}}{m}\nabla P(\rho, T_e) - \frac{q}{m}\nabla V = C_u, \tag{2.11}$$

$$\partial_t \mathcal{E} + \text{div}\,(u\mathcal{E} + uP(\rho, T_e)) - q\rho u \cdot \nabla V = C_{\mathcal{E}}. \tag{2.12}$$

The right hand side terms are the relaxation terms of momentum and temperature, respectively, stemming from the collision operator [64] (note that (2.9) does not annihilate (2.3)). When they are omitted, the system (2.10)–(2.12) becomes the Euler system of compressible charged particle flow in an electrostatic field. When applied to the semiconductor problem with a relaxation approximation of (2.3), the right hand side terms of (2.11) and (2.12) are given by

$$C_u = -\frac{\rho u}{\tau_u}, \quad C_{\mathcal{E}} = -\frac{1}{\tau_{\mathcal{E}}}\left(\frac{m}{2q^2}\rho|u|^2 + \frac{3}{2}\rho k_{\mathcal{B}}(T_e - T_L)\right),$$

where $\tau_u > 0$ and $\tau_{\mathcal{E}} > 0$ are the momentum and energy relaxation times respectively, and T_L is the lattice temperature. Moreover, a heat conduction term

$$-\text{div}\,(\mu\nabla T_e)$$

is often added to the left hand side of the energy equation (2.12).

Other approaches for the derivation of the hydrodynamical equations from the semiconductor Boltzmann equation were also given [4, 5, 72, 75]. E.g., in terms of the maximum entropy principle [66] the related problem of closure conditions was investigated [4, 6].

Now, rescaling the equations (2.10)–(2.12) and (2.6) and setting coefficients equal to one, we obtain the full hydrodynamical equations (FHD) for the electron mass, electron velocity and electron energy $(\rho, u, \mathcal{E})$

$$\partial_t \rho + \operatorname{div}(\rho u) = 0, \tag{2.13}$$

$$\partial_t (\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla p(\rho, T) = \rho \nabla V - \rho u, \tag{2.14}$$

$$\partial_t \mathcal{E} + \operatorname{div}(\mathcal{E} u + P(\rho, T)u - \mu \nabla T) = \rho u \cdot \nabla V - (\mathcal{E} - \mathcal{E}_L), \tag{2.15}$$

$$\Delta V = \rho - \mathcal{C}(x), \tag{2.16}$$

with the rest energy $\mathcal{E}_L \sim \frac{3}{2} \kappa \rho T_L$. Often the temperature is taken as a constant (or prescribed function of density) and we obain the so-called isentropic hydrodynamical model (HD)

$$\partial_t \rho + \operatorname{div}(\rho u) = 0, \tag{2.17}$$

$$\partial_t (\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla p = \rho \nabla V - \rho u, \tag{2.18}$$

$$\Delta V = \rho - \mathcal{C}(x), \tag{2.19}$$

with the pressure $p = p(\rho)$. A typical example is

$$p(\rho) = \frac{1}{\gamma} \rho^\gamma, \quad \gamma \geq 1. \tag{2.20}$$

Similarly, starting with the bipolar semiconductor Boltzmann equation for electrons and holes, and applying the moment method to the hole distribution function, we obtain the bipolar hydrodynamical model (BHD) [64] (here we only state the isentropic case)

$$\partial_t \rho_\alpha + \operatorname{div}(\rho_\alpha u) = 0, \tag{2.21}$$

$$\partial_t (\rho_\alpha u) + \operatorname{div}(\rho_\alpha u \otimes u) + \nabla p(\rho_\alpha) = \rho_\alpha \nabla V - \frac{\rho_\alpha u}{\tau_u}, \tag{2.22}$$

$$\partial_t \rho_\beta + \operatorname{div}(\rho_\beta u) = 0, \tag{2.23}$$

$$\partial_t (\rho_\beta v) + \operatorname{div}(\rho_\beta v \otimes v) + \nabla q(\rho_\beta) = -\rho_\beta \nabla V - \frac{\rho_\beta v}{\tau_v}, \tag{2.24}$$

$$\lambda^2 \Delta V = \rho_\alpha - \rho_\beta - \mathcal{C}(x), \tag{2.25}$$

where ρ_α, ρ_β, u, v are the densities and velocities for electrons and holes respectively. $\tau_u > 0$ and $\tau_v > 0$ are relaxation times for momentum, and λ is the Debye length. The mass and momentum equations for electrons and holes are coupled by the Poisson equation (2.25).

3 Asymptotic behavior of the unipolar HD

3.1 Subsonic classical solutions

In this subsection, we consider the initial boundary value problems (IVBP) for the isentropic hydrodynamical equations (2.17)–(2.19) in the one-dimensional x-interval $(0, 1)$ representing the semiconductor.

The initial data and the density and potential boundary values are given:

$$(\rho, j)(x, 0) = (\tilde{\rho}, \tilde{j})(x), \quad x \in (0, 1), \tag{3.1}$$

$$\rho(0, t) = \rho_1, \quad \rho(1, t) = \rho_2, \quad t \geq 0, \tag{3.2}$$

$$\phi(0, t) = 0, \quad \phi(1, t) = \phi_1, \quad t \geq 0. \tag{3.3}$$

We study the asymptotic behavior of solutions in the sense that we first consider the well-posedness of stationary solutions and then investigate their stability. The steady-state solutions of the hydrodynamic model for semiconductors satisfies the following boundary value problem (BVP):

$$j = \text{const}, \tag{3.4}$$

$$\left(\frac{j^2}{\rho} + p(\rho) \right)_x = \rho \phi_x - j, \tag{3.5}$$

$$\phi_{xx} = \rho - \mathcal{C}(x), \tag{3.6}$$

$$(3.2) \quad \text{and} \quad (3.3).$$

Here we set the current relaxation time to 1. For smooth solutions, the current-voltage relationship follows [16] by dividing (3.5) by ρ, and integrating over $[0, 1]$

$$\phi_1 = F(\rho_2, j) - F(\rho_1, j) + j \int_0^1 \frac{dx}{\rho(x)}, \tag{3.7}$$

where

$$F(\rho, j) = \frac{j^2}{2\rho^2} + h(\rho), \quad h'(\rho) = \frac{1}{\rho} p'(\rho). \tag{3.8}$$

Dividing (3.5) by ρ, differentiating, and using (3.4), (3.6), we obtain finally

$$\left(\frac{\partial F}{\partial \rho}(\rho, j)\rho_x\right)_x + j\left(\frac{1}{\rho}\right)_x - \rho = -\mathcal{C}(x), \quad 0 < x < 1. \tag{3.9}$$

Thus, to obtain the existence of regular solutions we require the subsonic condition to be satisfied:

$$\frac{\partial F}{\partial \rho}(\rho, j) = -\frac{j^2}{\rho^3} + \frac{1}{\rho}p'(\rho) > 0 \Leftrightarrow \rho^2 p'(\rho) > j^2. \tag{3.10}$$

Assuming that $\rho p'(\rho)$ is strictly monotonically increasing, we conclude that there is a unique $\rho_m = \rho_m(j)$ such that $\frac{\partial F}{\partial \rho}(\rho, j) > 0$ for $\rho > \rho_m$. Also, by (3.10), we know that the minimal point ρ_m of $\rho \to F(\rho, j)$ is a strictly increasing function of j with $\rho_m(j = 0) = 0$. These facts imply that the equation (3.9) is uniformly elliptic for $\rho \geq \rho^* > \rho_m$, which, by (3.10), means a fully subsonic flow $|u| < c(\rho)$. Here $c(\rho) = \sqrt{p'(\rho)}$ denotes the sound speed.

Assume that there is a function $\mathcal{A} = \mathcal{A}(x) \in C^2(0, 1)$ such that

$$\mathcal{A}(x) > 0, \ \mathcal{A}(0) = \rho_1, \ \mathcal{A}(1) = \rho_2, \ \mathcal{A} - \mathcal{C} \in C(0, 1). \tag{3.11}$$

For subsonic state, by the maximum value principle and Schauder's fixed point theorem, using the current-voltage relationship, we can prove the well-posedness of stationary solution (see [50] for details) as follows

Theorem 3.1 *Assume that $\rho p'(\rho)$ is strictly monotonically increasing, (2.20) and (3.11) hold. Let $J_0 \neq 0$ be such that*

$$\rho_1, \ \rho_2, \ \inf_{x \in (0,1)} \mathcal{C}(x) > \rho_m(J_0), \tag{3.12}$$

and assume $|\rho_2 - \rho_1| \ll 1$. Then there is a constant $\Phi_0 > 0$, such that for all $0 < \phi_1 \leq \Phi_0$, the BVP (3.4)–(3.6) and (3.2)–(3.3) has a unique solution (ρ_0, j_0, ϕ_0), which satisfies $|j_0| \leq |J_0|$ and

$$\mathcal{C}_* \stackrel{\Delta}{=} \min\{\rho_1, \ \rho_2, \ \inf_{x \in (0,1)} \mathcal{C}(x)\} \leq \rho_0(x) \leq \max\{\rho_1, \ \rho_2, \ \sup_{x \in (0,1)} \mathcal{C}(x)\} \stackrel{\Delta}{=} \mathcal{C}^*,$$

$$\|\rho_0 - \mathcal{A}\|_2^2 + \|\rho_{0x}\|_1^2 \leq C_0 \delta_0,$$

$$\|\phi_{0x}\|_1^2 \leq C_0 \delta_0,$$

where C_0 is a positive constant dependent on $\mathcal{C}_{\pm}$ and $|j_0|$, and

$$\delta_0 = \max_{x \in (0,1)} \{|\mathcal{A}'(x)| + |\mathcal{A}''(x)| + |\mathcal{A}(x) - \mathcal{C}(x)|\} + |\phi_1| + |\rho_2 - \rho_1|. \quad \square$$

Set

$$\psi_0 = \tilde{\rho} - \rho_0, \quad \eta_0 = \tilde{j} - j_0. \tag{3.13}$$

Standard local existence theorems for hyperbolic systems [56] and the a-priori estimates allow us to obtain the global existence of classical dynamical solutions and to study its large time behavior (see [50] for details):

Theorem 3.2. *Let* (ρ_0, j_0, ϕ_0) *be the regular solution of the BVP* (3.4)–(3.6) *and* (3.2)–(3.3) *given by Theorem 3.1. Assume* $(\psi_0, \eta_0) \in H^2$. *Then, there is* $\varepsilon_1 > 0$, *such that if* $\|(\psi_0, \eta_0)\|_2 + \delta_0 \leq \varepsilon_1$, *the global classical solution* (ρ, j, ϕ) *of the IBVP* (2.17)–(2.19) *and* (3.1)–(3.3) *exists and satisfies*

$$\begin{aligned} &\|(\rho - \rho_0, j - j_0, \phi - \phi_0)(\cdot, t)\|_2^2 \\ &\leq O(1)\|(\psi_0, \eta_0)\|_2^2 \exp\{-\Lambda_0 t\}, \quad t \geq 0 \end{aligned} \tag{3.14}$$

with a positive constant Λ_0. $\square$

Remark 3.3. The density and potential boundary values are realistic for semiconductor device simulation. Though analogous existence results of stationary solution in the multi-dimension compact domain were shown for the multidimensional isentropic HD [17, 76] and for the one-dimensional full HD [3], very few results were obtained on their stability for large time [79].

3.2   Subsonic shock solutions

In this subsection we consider the asymptotic behavior of subsonic entropy shock solutions of the initial value problem (*IVP*) for the hydrodynamical model (2.17) with the perturbed Riemann initial data:

$$(\rho, u)(x, 0) = (\tilde{\rho}, \tilde{u})(x) = \begin{cases} (\rho_l, u_l)(x), & x < 0, \\ (\rho_r, u_r)(x), & x > 0. \end{cases} \tag{3.15}$$

where

$$\lim_{x \to \pm\infty} (\tilde{\rho}, \tilde{u})(x) = (\mathcal{V}_\pm, \mathcal{U}_\pm), \quad \mathcal{V}_+\mathcal{U}_+ = \mathcal{V}_-\mathcal{U}_-, \tag{3.16}$$

and

$$(\varrho_-, u_-) = \lim_{x \to 0-} (\rho_l, u_l)(x) \neq \lim_{x \to 0+} (\rho_r, u_r)(x) =: (\varrho_+, u_+). \tag{3.17}$$

First, let us recall the basic shock theory of the compressible isentropic Euler equations [34, 15]

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho))_x = 0, \end{cases} \tag{3.18}$$

where the pressure p is given by (2.20). The equations (3.18) can be written as a system

$$\mathbf{v}_t + f(\mathbf{v})_x = 0, \tag{3.19}$$

where $\mathbf{v} = (\rho, j)^T$ with $j = \rho u$, $f(\mathbf{v}) = (j, \frac{j^2}{\rho} + p(\rho))^T$. The Jacobi matrix of f is

$$\mathbf{D}f = \begin{pmatrix} 0 & 1 \\ -\frac{j^2}{\rho^2} + p'(\rho) & \frac{2j}{\rho} \end{pmatrix}. \tag{3.20}$$

The eigenvalues of (3.20) are

$$\lambda_1 = \frac{j}{\rho} - \sqrt{\gamma}\rho^{(\gamma-1)/2}, \quad \lambda_2 = \frac{j}{\rho} + \sqrt{\gamma}\rho^{(\gamma-1)/2}, \tag{3.21}$$

or

$$\lambda_1 = u - \sqrt{\gamma}\rho^{(\gamma-1)/2}, \quad \lambda_2 = u + \sqrt{\gamma}\rho^{(\gamma-1)/2}. \tag{3.22}$$

A i-shock wave, $i = 1, 2$, for (3.19) is characterized by the Rankine-Hugoniot condition and Lax entropy condition. Namely, along the discontinuity $x = x_i(t)$ it holds

$$\begin{cases} \dot{x}_1(t) = -\sqrt{[\rho u^2 + p(v)]_1/[\rho]_1}, \\ [\rho u]_1 = \sqrt{[\rho u^2 + p(\rho)]_1/[\rho]_1} \cdot [\rho]_1, \\ \lambda_1(x_1(t) - 0, t) > \dot{x}_1(t) > \lambda_1(x_1(t) + 0, t), \end{cases} \tag{3.23}$$

or

$$\begin{cases} \dot{x}_2(t) = \sqrt{[\rho u^2 + p(\rho)]_2/[\rho]_2}, \\ [\rho u]_2 = -\sqrt{[\rho u^2 + p(\rho)]_2/[\rho]_2} \cdot [\rho]_2, \\ \lambda_2(x_2(t) - 0, t) > \dot{x}_2(t) > \lambda_2(x_2(t) + 0, t), \end{cases} \tag{3.24}$$

where we denote

$$[F]_i = F(x_i(t) + 0, t) - F(x_i(t) - 0, t), \quad i = 1, 2.$$

Denote the electric field $E = \phi_x$, then the hydrodynamical equations (2.17)–(2.19) can be written as

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p(\rho))_x = \rho E - \rho u, \\ E_x = \rho - \mathcal{C}(x), \end{cases} \tag{3.25}$$

For simplicity, we consider the IVP (3.25) and (3.15) in the case that the two states (ϱ_-, u_-) and (ϱ_+, u_+) are connected by two shock curves in phase space, i.e., there is a state (ϱ_c, u_c) such that

$$\begin{cases} u_- > u_c > u_+, \\ (\varrho_c u_c - \varrho_- u_-)^2 = (\varrho_c u_c^2 - \varrho_- u_-^2 + p(\varrho_c) - p(\varrho_-)) \\ \qquad\qquad (\varrho_c - \varrho_-), \varrho_- < \varrho_c, \\ (\varrho_+ u_+ - \varrho_c u_c)^2 = (\varrho_+ u_+^2 - \varrho_c u_c^2 + p(\varrho_+) - p(\varrho_c)) \\ \qquad\qquad (\varrho_+ - \varrho_c), \varrho_+ < \varrho_c \end{cases} \tag{3.26}$$

We also expect that there exists a stationary solution of (3.25) which is the asymptotic state of the dynamical solutions. Let us consider the stationary solutions $(\rho_0, j_0, E_0)(x)$ of (3.25) with boundary value

$$\rho_0 E_0(\pm\infty) = \mathcal{C}_\pm E_\pm = j_0, \tag{3.27}$$

where $\mathcal{C}_\pm = \mathcal{C}(\pm\infty) > 0$. We have

Theorem 3.4 [57]. *Assume that (2.20) holds and* $\mathcal{C} \in H^1 \cap L^1 \cap W^{1,4}$. *Let* j_0 *be such that*

$$\min_{x \in \mathbb{R}} \mathcal{C}(x) > \rho_m(j_0).$$

Then, there exists a unique (up to a shift) smooth solution (ρ_0, j_0, E_0) *of BVP (3.25), (3.27) such that*

$$\mathcal{C}_* \overset{\Delta}{=} \min_{x \in \mathbb{R}} \mathcal{C}(x) \leq \rho_0(x) \leq \mathcal{C}^* \overset{\Delta}{=} \max_{x \in \mathbb{R}} \mathcal{C}(x),$$

$$|\rho_0(x) - \mathcal{C}_\pm| = O(e^{-c_\pm |x|}) \text{ as } x \to \pm\infty,$$

$$\|\tilde{\rho} - \mathcal{C}\|_2^2 + \sup_{x \in R}(|\rho_0'(x)|^2 + |\rho_0''(x)|^2) \leq C_0\delta_1,$$

$$\sup_{x \in R} |E_0(x)|^2 \leq C_0(\delta_1 + \delta_2),$$

where

$$c_\pm = \frac{E_\pm}{p'(\mathcal{C}_\pm) - E_\pm^2} > 0,$$

$$\delta_1 = |\log \mathcal{C}_+ - \log \mathcal{C}_-| + \|\mathcal{C}'\|_{L^1} + \|\mathcal{C}'\|_{H^1}^2 + \|\mathcal{C}'\|_{L^4}^4, \quad \delta_2 = \rho_m^2 p'(\rho_m)/\mathcal{C}_*^2,$$

and $C_0 > 0$ is a constant only depending on $\mathcal{C}$. □

Set

$$\psi_1(x) = \left\{ \int_{-\infty}^{0-} + \int_{0+}^{x} \right\} (\tilde{\rho}(y) - \rho_0(y)) dy, \quad \eta_1(x) = \tilde{\rho}\tilde{u}(x) - j_0.$$

Denote

$$\delta_3 = |\varrho_c - \varrho_+| + |\varrho_- - \varrho_c|, \quad \mu_0 = \delta_1 + \delta_2,$$

$$\mu_1 = \|(\varepsilon_0, \varepsilon_{0x}, \varepsilon_{0xx}, \varepsilon_{0xxx})\| + \|(\varepsilon_1, \varepsilon_{1x}, \varepsilon_{1xx})\| < +\infty, \tag{3.28}$$

$$\mu_2 = \sum_{i=0}^{2} \sup_{x \neq 0} \left\{ |\partial_x^i(\tilde{\rho} - \rho_0)| + |\partial_x^i(\tilde{u} - u_0)| \right\} < +\infty, \tag{3.29}$$

$$\mu_3 = \sup_{x \neq 0} \left\{ |\partial_x^3(\tilde{\rho} - \rho_0)(x)| + |\partial_x^3(\tilde{u} - u_0)(x)| \right\} < +\infty, \tag{3.30}$$

where

$$\| f \| = \sqrt{ \int_{-\infty}^{0-} |f(x)|^2 dx + \int_{0+}^{\infty} |f(x)|^2 dx }, \tag{3.31}$$

$\rho_0 u_0(x) = j_0$. We have the following main result

Theorem 3.5. *Let $(\rho_0, u_0) \in C^3(R - \{0\})$, $\psi_1 \in L^2(R)$, and $(\psi_1, \varepsilon_1) \in H^3(R - \{0\}) \times H^2(R - \{0\})$. Let (3.26) and (3.28)–(3.30) hold. Then there exists $\varepsilon_2 > 0$ such that if $\delta_3 + \mu_0 + \mu_1 + \mu_2 < \varepsilon_2$, the global entropy weak solution (ρ, u, E) of the IVP (3.25) and (3.15) exists. It is piecewise continuous and piecewise smooth with two shock discontinuities – a forward shock curve $x = x_2(t)$ and a backward shock curve $x = x_1(t)$ satisfying $x_1(0) = x_2(0) = 0$ and $x_1(t) < x_2(t)$ for $t > 0$. Away from the discontinuities, $(\rho, u, E_x)(\cdot, t) \in C^3$. In addition, as t tends to infinity,*

$$\sum_{i=0}^{2} \left(|[\partial_x^i(\rho, u)]_1| + |[\partial_x^i(\rho, u)]_2| \right) \sim O(1) e^{-\Lambda_1 t} \to 0, \tag{3.32}$$

and

$$\sum_{i=0}^{2} \|\partial_x^i (\rho - \rho_0, \rho u - j_0, E - E_0)(\cdot, t)\| \sim O(1)e^{-\Lambda_2 t} \to 0, \qquad (3.33)$$

with two positive constants Λ_1 and Λ_2.   □

Key point of proof. One construct the piecewise smooth solution based on the local geometric structure of solutions of the Riemann problem for inhomogeneous hyperbolic equations [52] which says that the discontinuous initial value problem (3.25) and (3.15) admits a unique discontinuous solution in short time in the class of piecewise continuous and piecewise smooth functions with a forward shock $x = x_2(t)$ and a backward shock $x = x_1(t)$ both passing through $(0, 0)$.

To extend the local structure globally in time, we need to study the geometry of shock structure and show that the shock strength decays as time grows up but the shocks never disappear in finite time. Moreover, away from the discontinuities we have to control the oscillations of solutions so as to prevent new singularities from formation. This can be done by solving the three independent free boundary value problems for (3.25) separated by the boundary $x = x_1(t)$ and $x = x_2(t)$ in terms of the characteristic methods and the (relaxation) damping dissipations. The Lax-entropy conditions guarantee the well-posedness of the corresponding free boundary value problems. The large time behavior is then obtained in terms of energy methods, which basically is similar to that for the classical solutions except that one uses the piecewise energy estimates here. The reader is refered to [51] for details.

4   Dynamic behavior of the bipolar HD

The Cauchy problem for one-dimensional bipolar HD model without doping reads:

$$\partial_t \rho_\alpha + (\rho_\alpha u)_x = 0, \qquad (4.1)$$

$$\partial_t (\rho_\alpha u) + (\rho_\alpha u^2 + p(\rho_\alpha))_x = \rho_\alpha E - \frac{\rho_\alpha u}{\tau_u}, \qquad (4.2)$$

$$\partial_t \rho_\beta + (\rho_\beta v)_x = 0, \qquad (4.3)$$

$$\partial_t (\rho_\beta v) + (\rho_\beta v^2 + q(\rho_\beta))_x = -\rho_\beta E - \frac{\rho_\beta v}{\tau_v}, \qquad (4.4)$$

$$\lambda^2 E_x = \rho_\alpha - \rho_\beta. \qquad (4.5)$$

with initial data:

$$(\rho_\alpha, u, \rho_\beta, v)(x, 0) = (\tilde{\rho}_\alpha, \tilde{u}, \tilde{\rho}_\beta, \tilde{v})(x)$$
$$\longrightarrow (\rho_\pm, 0, \rho_\pm, 0) \text{ as } x \to \pm\infty. \tag{4.6}$$

Here $E_-(t)$, the electric filed at $x = -\infty$, is assumed to be given and set to zero without the loss of generality.

Since the Poisson coupling (4.5) is somewhat weaker without the background ions, the expected asymptotic state of the classical solutions $(\rho_\alpha, u\rho_\beta, v, E)$ may be different from those in the previous section due to the (relaxation) damping dissipations. In fact, since it is known that solutions of the damped Euler equations behave like nonlinear diffusive waves in large time [35, 36], we may expect that both the electrons and holes density will tend to nonlinear diffusive waves. Let us understand roughly this phenomena by applying the quasi-neutral limit [28]. Set $\tau_u = \tau_v = \tau$ in (4.1)–(4.5) for simplicity and re-scale

$$t \to \tau t, \ \lambda^2 = \tau^{1+s}, \ -1 < s,$$
$$\rho_\alpha^\tau = \rho_\alpha(\frac{t}{\tau}, x), \ u^\tau = \frac{1}{\tau}u(\frac{t}{\tau}, x),$$
$$\rho_\beta^\tau = \rho_\beta^\tau(\frac{t}{\tau}, x), \ v^\tau = \frac{1}{\tau}v(\frac{t}{\tau}, x),$$

we obtain the scaled equations:

$$\partial_t \rho_\alpha^\tau + (\rho_\alpha^\tau u^\tau)_x = 0, \tag{4.7}$$

$$\tau^2 \partial_t(\rho_\alpha^\tau u^\tau) + (\tau^2 \rho_\alpha^\tau (u^\tau)^2 + p(\rho_\alpha^\tau))_x = \rho_\alpha^\tau E^\tau - \rho_\alpha^\tau u^\tau, \tag{4.8}$$

$$\partial_t \rho_\beta^\tau + (\rho_\beta^\tau v^\tau)_x = 0, \tag{4.9}$$

$$\tau^2 \partial_t(\rho_\beta^\tau v^\tau) + (\tau^2 \rho_\beta^\tau (v^\tau)^2 + q(\rho_\beta^\tau))_x = -\rho_\beta^\tau E^\tau - \rho_\beta^\tau v^\tau, \tag{4.10}$$

$$\tau^{1+s} E_x^\tau = \rho_\alpha^\tau - \rho_\beta^\tau. \tag{4.11}$$

Let $\tau \to 0$ in (4.7)–(4.11), we obtain the equation for the limiting densities of ρ_α^τ and ρ_β^τ:

$$w_t = \frac{1}{2}(p(w) + q(w))_{xx}, \tag{4.12}$$

with $\rho_\alpha = \rho_\beta = w$. In the following analysis, we just take

$$p(\rho) = q(\rho) = \frac{1}{\gamma}\rho^\gamma, \quad \gamma \geq 1. \tag{4.13}$$

As shown in [23] the nonlinear equation (4.12) admits a unique self-similar solution $w(x, t) = W(\xi)$, $\xi = \frac{x}{\sqrt{1+t}}$, satisfying

$$W(\pm\infty) = \rho_\pm. \tag{4.14}$$

Finally, we prove rigorously [29] that the dynamical solutions of the IVP (4.1)–(4.6) convergence to this self-similar solution $W(\xi)$ as time tends to infinity. For the role of simplicity we set the Debye length and both the relaxation times to 1.

Set

$$\psi_2 = \int_{-\infty}^{x} (\tilde{\rho}_\alpha(y) - W(y + x_0))dy, \quad \eta_2 = \tilde{\rho}_\alpha\tilde{u}(x) + p(W(x + x_0))_x, \tag{4.15}$$

$$\psi_3 = \int_{-\infty}^{x} (\tilde{\rho}_\beta(y) - W(y + y_0))dy, \quad \eta_3 = \tilde{\rho}_\beta\tilde{v}(x) + p(W(x + y_0))_x, \tag{4.16}$$

with x_0 and y_0 chosen such that

$$\psi_2(+\infty) = 0, \; \psi_3(+\infty) = 0.$$

Then we obtain

Theorem 4.1. *Assume that (4.13) holds. Suppose that $(\psi_2, \eta_2, \psi_3, \eta_3) \in H^3 \times H^2 \times H^3 \times H^2$ with $x_0 = y_0$. Then, there is $\varepsilon_3 > 0$ such that if $\|(\psi_2, \psi_3)\|_3 + \|(\eta_2, \eta_3)\|_2 \le \varepsilon_3$, the global classical solution $(\rho_\alpha, u, \rho_\beta, v, E)$ of IVP (4.1)– (4.5) and (4.6) exists and satisfies*

$$\|(\rho_\alpha - W, \rho_\beta - W)(., t)\|_2 \le C(1 + t)^{-1/2}, \tag{4.17}$$

$$\|(\rho_\alpha - \rho_\beta, E)(., t)\|_2 \le Ce^{-\Lambda_3 t}, \tag{4.18}$$

with $\Lambda_3 > 0$ a constant.

Furthermore, if we assume in addition $(\psi_2, \psi_3) \in L^1$, we have the following (optimal) L_p $(2 \le p \le \infty)$ decay rates:

$$\|\partial_x^k(\rho_\alpha - W, \rho_\beta - W)(., t)\|_{L_p} \le C|\rho_+ - \rho_-|(1 + t)^{-\frac{1}{2}(1 - \frac{1}{p}) - \frac{k+1}{2}}, \tag{4.19}$$

$$\|\partial_x^k(u + p(W)_x, v + p(W)_x)(., t)\|_{L_p} \le C|\rho_+ - \rho_-|(1 + t)^{-\frac{1}{2}(1 - \frac{1}{p}) - \frac{k+2}{2}}, \tag{4.20}$$

for $k \le 2$ if $p = 2$ and $k \le 1$ if $p \in (2, \infty]$. $\square$

Key point of proof.   The results are proven by using energy estimates for the damped wave equations obtained from the perturbations of the equations for electrons and holes, and for the damped Klein-Gordon equation for electric field. The reader is refered to [29] for details.

Remark 4.2. The above time-asymptotic behavior of the bipolar HD model (4.10–(4.5) is not surprising. In fact, such diffusive phenomena also occur in the quasineutrual limit of the bipolar HD model [28] even in dimension larger than 1.   □

References

[1]   G. Ali, D. Bini and S. Rionero, *Hydrodynamic model for semiconductors with varying temperature: global existence of smooth solutions*, SIAM J. Math. Anal., **32** (2000), 572–587.

[2]   N. Aluru, K. Law, P. Pinsky and R. Dutton, *An analysis of the hydrodynamic semiconductors device model–boundary conditions and simulations*, Compel, **14** (1995), 157–185.

[3]   P. Amster, M.P. Beccar Varela, A. Jüngel and M.C. Mariani, *Subsonic solutions to a one-dimensional non-isentropic hydrodynamic model for semiconductors*, J. Math. Anal. Appl., to appear.

[4]   A. Anile and O. Muscato, *Improved hydrodynamical model fro carrier transport in semiconductors*, Phys. Rev. B, **51** (1995), 16728–16740.

[5]   A. Anile and O. Muscato, *Extended thermodynamics tested beyond the linear regime: the case of electron transport in semiconductors*, Cont. Mech. Thermodyn., **8** (1996), 131–142.

[6]   A. Anile and A. Romano, *Nonparabolic band in semiconductors: closure of the moment equations*, Cont. Mech. Thernodyn.

[7]   U. Ascher, P.A. Markowich and C. Schmeiser, *A phase plane analysis of transonic solutions for the hydrodynamic semiconductor model*, Math. Models Meth. Appl. Sci., **1**, (1991), 347–376.

[8]   N. Ashcroft and N. Mermin, *Solid Sstate Physics*, Sauners College, Philadelphia, 1976.

[9]   K. Bløtekjær, *Transport equations for electrons in two-valley semiconductors*, IEEE Trans. Electron Devices, ED-17, 38–47 (1970).

[10]   G.-Q. Chen, *Convergence of Lax-Friedrichs scheme for isentropic gas dyamics III*, Acta. Math. Sci., **6** (1986), 75–120.

[11] G.-Q. Chen, J. Jerome and B. Zhang, *Existence and the singular relaxation limit for the inviscid hydrodynamic energy model*, Numerical Mathematics and Scientific Computation, Jerome, Joseph W. (ed.), Oxford: Clarendon Press, 189–215 (1998).

[12] G.-Q. Chen, J. Jerome and B. Zhang, *Particle hydrodynamic moment models in biology and microelectronics: singular relaxation limits*, Nonlinear Anal., TMA, **30** (1997), 233–244.

[13] G.-Q. Chen and D. Wang, *Convergence of shock schemes for the compressible Euler-Poisson equations*, Comm. Math. Phys., **179** (1996), 333–364.

  1. CW98 G.-Q. Chen and D. Wang, *Formation of singularities in compressible Euler-Poisson fluids with heat diffusion and damping relaxation*, J. Differential Equations, **144** (1998), 44–65.

[15] C.M. Dafermos, *Hyperbolic conservation laws in continuum physics*, Springer-Verlag, Berlin 2000.

[16] P. Degond and P.A. Markowich, *On a one-dimensional steady-state hydrodynamic model*, Appl. Math. Lett., **3** (1990), 25–29.

[17] P. Degond and P.A. *Markowich, A steady-state potential flow model for semiconductors*, Ann. Math. Pure Appl., **IV** (1993), 87–98.

[18] X. Ding, G.-Q. Chen and P. Luo, *Convergence of the Lax-Friedrichs scheme for isentropic gas dynamics I*, Acta Math. Sci., **5** (1985), 415–432.

[19] X. Ding, G.-Q. Chen and P. Luo *Convergence of the Lax-Friedrichs scheme for isentropic gas dynamics II*, Acta Math. Sci., **5** (1985), 433–472.

[20] X. Ding, G.-Q. Chen and P. Luo *Convergence of the fractional step Lax-Friedrichs scheme and Godunov scheme for isentropic gas dynamics*, Comm. Math. Phys., **121** (1989), 63–84.

[21] R.J. DiPerna *Convergence of the viscosity method for isentropic gas dynamics*, Comm. Math. Phys., **91** (1983), 1–30.

[22] R.J. DiPerna *Convergence of approximate solutions to conservation laws*, Arch. Rat. Mech. Anal., **82** (1983), 27–70.

[23] C.T. Duyn and L.A. Peletier, *A class of similarity solutions of the nonlinear diffusion equation*, Nonlinear Analysis, TMA, **1** (1977), 223–233.

[24] W. Fang and K. Ito, *Weak solutions to a one-dimensional hydrodynamic model for semiconductors*, Nonlinear Analysis, TMA, **28** (1997), 947–963.

[25] W. Fang and K. Ito, *Steady-state solutions of a one-dimensional hydrodynamic model for semiconductors*, J. Differential Equations, **133** (1997), 224–244.

[26] I. Gamba, *Stationary transonic solutions of a one-dimensional hydrodynamic model for semiconductor*, Comm. Partial Diff. Equns, **17** (3/4) (1992), 553–577.

[27] I. Gamba and C.S. Morawetz, *A Viscous approximation for a 2-D steady semiconductor or transonic gas dynamic flow: existence theorem for potential flow*, Comm. Pure Appl. Math., **49** (1996), 999–1049.

[28] I. Gasser, *A review on small debye length and quasineutral limits in macroscopic models for charged fluids*, preprint.

[29] I. Gasser, L. Hsiao and H.-L. Li, *Asymptotic behavior of solutions of bipolar hydrodynamical fluids*, preprint.

[30] I. Gasser and R. Natalini, *The energy transport and the drift diffusion equations as relaxation limits of the hydrodynamic model for semiconductors*, Quart. Appl. Math., **57** (1996), 269–282.

[31] H. Hattori, *Stability and instability of steady-state solutions for a hydrodynamic model of semiconductors*, Proc. Roy. Soc. Edinburgh A, **127** (1997), 781–796.

[32] H. Hattori and C. Zhu, *Asymptotic behavior of the solutions to a non-isentropic hydrodynamic model of semiconductors*, J. Differential Equations, **144** (1998), 353–389.

[33] H. Hattori and C. Zhu, *Stability of steady state solutions for an isentropic hydrodynamic model of semiconductors of two species*, J. Differential Equations, **166** (2000), 1–32.

[34] L. Hsiao and T. Zhang, *The Riemann problem and interaction of waves in gas dynamics*, Longmann Scientific and Technical, 1989.

[35] L. Hsiao and T.-P. Liu, *Nonlinear diffusive phenomena of nonlinear hyperbolic systems*, Chin. Ann. Math., **14B** (1993), 465–480.

[36] L. Hsiao and T.-P. Liu, *Convergence to nonlinear diffusive waves for solutions of a system of hyperbolic conservation laws with damping*, Comm. Math. Phys., **143** (1992), 599–605.

[37] L. Hsiao and S. Wang, *The asymptotic behavior of global smooth solutions to the macroscopic models for semiconductors*, Chin. Ann. of Math., **22B** (2001), 195–210.

[38] L. Hsiao, P. Markowich and S. Wang, *Asymptotic behavior of globally smooth solutions to the multidimensional isentropic hydrodynamic model for semiconductors*, preprint.

[39] L. Hsiao, S. Wang and H.J. Zhao, *Asymptotic behavior of global solutions to the multidimensional hydrodynamic model for semiconductors*, Math. Method in the Applied Sciences, to appear.

[40] L. Hsiao and T. Yang, *Asymptotic of initial boundary value problems for hydrodynamic and drift diffusion models for semiconductors*, J. Differential Equations, **170** (2001), 472–493.

[41] L. Hsiao and K. Zhang, *The relaxation of the hydrodynamic model for semiconductors to drift diffusion equations*, J. Differential Equations, **165** (2000), 315–354.

[42] L. Hsiao and K. Zhang, *The global weak solution and relaxation limits of the initial boundary value problem to the bipolar hydrodynamic model for semiconductors*, Math. Models Methods Appl. Sci., **10** (2000), 1333–1361.

[43] J. Jerome, *Analysis of charge transport: a mathematical study of semiconductor devices*, Springer-Verlag, Heidelberg (1996).

[44] F. Jochmann, *Global weak solutions of the one-dimensional hydrodynamical model for semiconductors*, Math. Models Meth. Appl. Sci., **3** (1993), 759–788.

[45] A. Jüngel, *Quasi-hydrodynamic semiconductor equations*, Progress in Nonlinear Differential Equations, Birkhäuser, 2001.

[46] A. Jüngel, *Macroscopic models for semiconductor devices: a review*, preprint.

[47] A. Jüngel and Y.J. Peng, *Zero-relaxation-time limits in the hydrodynamic equations for plasmas revisited*, Z. Angew. Math. Phys., **51** (2000), 385–396.

[48] A. Jüngel and Y.J. Peng, *A hierachy of hydrodynamic models for plasmas: Zero-relaxation-time limits*, Comm. P. D. E., **24** (1999), 1007–1033.

[49] P.D. Lax, *Hyperbolic systems of conservation laws and the mathematical theory of shock waves*, Philadelphia, SIAM, 1973.

[50] H.-L. Li, P. Markowich and M. Mei, *Asymptotic Behavior of Solutions of the Hydrodynamic Model of Semiconductors*, Proc. Royal Soci. Edinburgh A, to appear.

[51] H.-L. Li, P. Markowich and M. Mei, *Asymptotic Behavior of subsonic shock solutions of the isentropic Euler-Poisson equations*, Quarterly Appl. Math., to appear.

[52] T.T. Li and W.C. Yu, *Boundary value problem for quasilinear hyperbolic systems*, Duke Univ. Math., Ser. V, 1985.

[53] P.L. Lions, B. Perthame and E. Tadmor, *Kinetic formulation of the isentropic gas dynamics and p-systems*, Comm. Math. Phys., **163** (1994), 415–431.

[54] P.L. Lions, B. Perthame and E. Sougandis, *Existence of entropy solutions for the hyperbolic systems of isentropic gas dynamics in Eulerian and Lagrangian coordinates*, Comm. Pure Appl. Math., **44** (1996), 599–638.

[55] T. Luo, R. Natalini and Z. Xin, *Large time behavior of the solutions to a hydrodynamic model for semiconductors*, SIAM J. Appl. Math., **59** (1998), 810–830.

[56] A. Majda, *Compressible Fluid Flow and Systems of Conservation Laws in Several Space Variables*, Appl. Math. Sci., **53** (1984), Springer-Verlag.

[57] P. Marcati and M. Mei, *Convergence to steady-state solutions of the initial boundary value problem to a hydrodynamic model for semiconductors*, Quarterly Appl. Math., **58** (2000), 763–784.

[58] P. Marcati and R. Natalini, *Weak solutions to a hydrodynamic model for semiconductors: the Cauchy problem*, Proc. Royal Soc. Edinburgh A, **125** (1995), 115–131.

[59] P. Marcati and R. Natalini, *Weak solutions to a hydrodynamic model for semiconductors and relaxation to the drift-diffusion equation*, Arch. Rational Mech. Anal., **129** (1995), 129–145.

[60] P.A. Markowich, *The Steady-State Semiconductor Device Equations*, Springer, Wien, New York, (1986).

[61] P.A. Markowich, *On steady-state Euler-Poisson model for semiconductors*, Z. Ang. Math. Phys., **62** (1991), 389–407.

[62] P.A. Markowich and C. Schmeiser, *The drift-diffusion limit for electron-photon interaction in semiconductors*, Math. Models Methods Appl. Sci., **7** (1997), 707–729.

[63] P.A. Markowich and P. Pietra, *A non-isentropic Euler-Poisson model for a collisionless plasma*, Math. Methods Appl. Sci., **16** (1993), 409–442.

[64] P.A. Markowich, C. Ringhofer and C. Schmeiser, *Semiconductor Equations*, Springer, Wien, New York, (1989).

[65] N. Mauser, Y. Qiu and K. Zhang, *Global existence and asymptotic limits of weak solutions on the bipolar hydrodynamic model for semiconductors*, preprint.

[66] I. Müller and T. Ruggeri, *Rational Extended Thermodynamics*, Springer, New-York, 1998.

[67] R. Natalini, *The bipolar hydrodynamic model for semiconductors and the drift-diffusion equations*, J. Math. Anal. Appl., **198** (1996), 262–281.

[68] F. Poupaud, *On a system of nonlinear Boltzmann equations of semiconductor physics*, SIAM J. Appl. Math., **50** (1990), 1593–1606.

[69] F. Poupaud, *Diffusion approximation of the linear semiconductor Boltzmann equation: analysis of boundary layer*, Asymptotic analysis, **4** (1991), 293–317.

[70] F. Poupaud, M. Rascle and J.-P. Vila, *Global solutions to the isothermal Euler-Poisson system with arbitrarily large data*, J. Differential Equations, **123** (1995), 93–121.

[71] W.V. van Roosbroeck, *Theory of electrons and holes in Germanium and other semiconductors*, Bell Syst. Techn. J., **29** (1950), 560–607.

[72] M. Rudan, A. Gnudi and W. Quaade, *A generalized approach to the hydrodynamic model of semiconductor equations*, Process and device modeling for microelectronics, Amsterdam, Elsevier, 1993.

[73] E. Thomann and F. Odeh, *On the well-posedness of the two-dimensional hydrodynamic model for semiconductor devices*, Compel, **9** (1990), 45–57.

[74] S. Selberherr, *Analysis and Simulation of Semiconductor Device Equations*, Springer, Wien, New York, (1984).

[75] G. Ulich and K. Asano, *Existenz einer lösung für hydrodynamishchesmodell aus der halbleiterphysik*, Diploma thesis, TU-Berlin, Gernamy, 1991.

[76] L. Yeh, *Subsonic solutions of hydrodynamic model for semiconductors*, Math. Methods Appl. Sci., **20** (1997), 1389–1410.

[77] B. Zhang, *Convergence of the Godunov scheme for a simplified one-dimensional hydrodynamic model for semiconductor devices*, Comm. Math. Phys., **157** (1993), 1–22.

[78] B. Zhang, *On a local existence theorem for a simplified one-dimensional hydrodynamic model for semiconductor devices*, SIAM J. Math. Anal., **25** (1994), 941–947.

[79] B. Zhang, *Global existence and asymptotic stability to the full 1D hydrodynamic model for semiconductor devices*, Indiana Univ. Math. J., **44** (1995), 971-1005.

[80] K. Zhang, *Global weak solutions of the Cauchy problem to a hydrodynamic model for semiconductors*, J. Partial Differential Equations, **12** (1999), 369–383.

[81] K. Zhang, *On the initial-boundary value problem for the bipolar hydrodynamic model for semiconductors*, J. Differential Equations, **171** (2001), 257–293.

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